# Finite Element Method Based Analysis of Planetary Gear Systems Considering Backlash and Manufacturing Deviations

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The load carrying capacity of gear transmissions depends strongly on design, material and operation conditions. Modern analysis methods, e.g. — finite element analysis (FEA) — consider the above parameters with more or less sufficient accuracy. Yet it remains an ongoing challenge to account for backlash and manufacturing errors, despite a definite need to do so. There are in fact several difficulties to overcome; first, the contact footprints as well as backlash and errors are some orders of magnitude smaller than the dimensions of the gears. Therefore an extremely fine mesh is required. On the other hand, the total number of degrees of freedom (DOFs) should be kept as small as possible to keep the computational effort manageable. Less stressed areas should be coarsely meshed, while the transition must be as smooth as possible. Moreover, in order to generate and mesh three-dimensional tooth flanks taking into consideration the abovementioned geometrical deviations is extremely time consuming.

In this paper new techniques and tools are introduced, aiming to facilitate the modelling procedure. They include automatic generation of 3-D gear flanks and selective meshing. A simple planetary gear system featuring helical gears with backlash and manufacturing errors is used to demonstrate the effectiveness of the proposed procedure, since it features multiple contacts and floating members. Results shown include stress distribution and deformation, as well as load sharing under quasi-static conditions.

Planetary gear trains offer very high power density because of their internal power splitting. It is evident that manufacturing deviations have a strong impact on power distribution. Therefore simulations aiming at the prediction of the actual loading of real, non-ideal, planetary systems are very useful. A lot of research has been conducted in this field. Reference 1 contains an extensive literature review. Finite element analysis is, of course, one of the first available methods to be used. It should be considered that an adequate mesh must be generated to capture the influence of inaccuracies. This is quite challenging because of several reasons. For example, the gears themselves are quite complex and the deviations are some orders of magnitude smaller than the overall dimensions of the complete system. In a previous paper (Ref. 1), the influence of backlash and manufacturing errors of carrier and gears of a simple planetary system is demonstrated. Furthermore, the favorable impact of a floating, self-centering sun on the load distribution was shown. However, the model was 2-dimensional; this is an important drawback, as it restricts its application to straight spur gears. The current study advances one step further and introduces a 3-D model that makes possible the analysis of planetary systems consisting of helical gears.

## Planetary Gear Train

In the current study a simple planetary gear system with helical gears is modeled (Fig. 1); it consists of a fixed ring gear mounted on the housing, a carrier with three equally spaced planets, and the sun. Two versions are studied; in the first, the sun rotates around a fixed axis or, put in another way, the input shaft is supported by two bearings; in the second version the input torque is transmitted to the sun gear by a double-articulated spline coupling. Therefore it is free to position itself between the planet gears. The planet gears are mounted on the pins by two roller bearings. The carrier is mounted in the housing also by two bearings. Torque

| | Sun | Planets | Ring |
|---|---|---|---|
| Number of teeth z | 13 | 24 | −65 |
| Addendum modification coefficient x | 0.8 | 0.757 | 0.1017 |
| Tip diameter $d_a$ [mm] | 28.65 | 16.85 | 67.6 |
| Module $m_n$ [mm] | | 1 | |
| Helix angle β [degrees] | | 20 | |
| Centre distance a [mm] | | 21 | |
| Facewidth b [mm] | | 10 | |
| Bottom clearance $c_p$ [mm] | | 0.25 | |
| Basic rack | | DIN 867 | |

**Figure 1 Gear data and simple planetary gear train with helical gears.**

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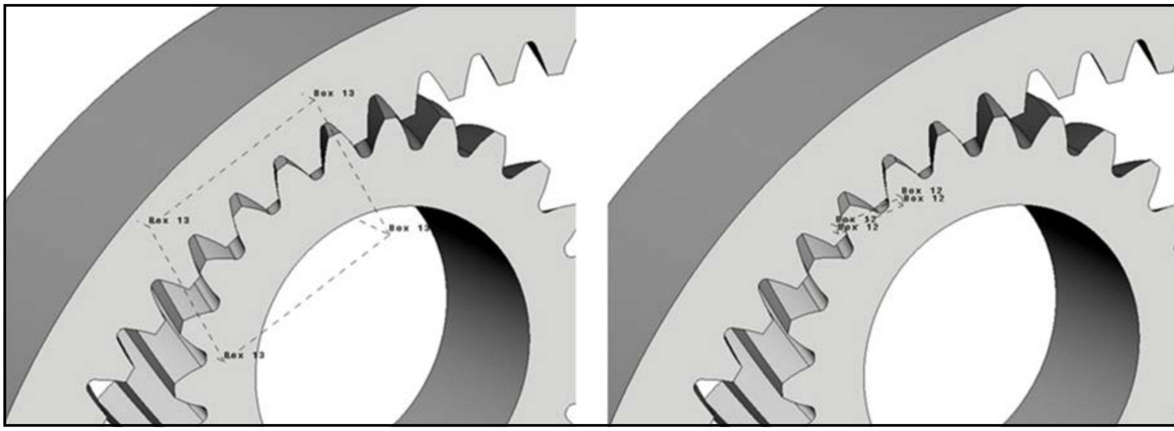


Figure 2 Refinement boxes at planet — ring gear mesh; (a) root fillet; (b) plane of action.

output is through the carrier. The carrier and the pins are considered very stiff compared to the gears, and therefore are not included in the model. All gears have the same facewidth and are made from case hardened steel; the gear data are included (Fig. 1).

## Finite Element Model

At first, a geometrically perfect gear tooth profile is generated by rolling a rack of given geometry on the pitch circle of the gear. It is then simultaneously moved along the gear axis and rotated around it. In this way the helical flanks of a tooth are produced. The complete gear is then built as a circular pattern of the single tooth. Following this procedure, geometrically perfect gears are created. Backlash can be introduced to the gears by properly decreasing the profile shift.

The next task is to generate the mesh, which is done in three steps. In step 1 the 3-D surfaces of the gears are meshed using triangular shell (2-D) elements; in step 2, solid (tetrahedral) elements are automatically created in the volume defined by the previously created shell elements. The resulting mesh is unstructured. In the third step manufacturing deviations are imposed in the model.

The main problem that needs to be addressed is how to make the mesh as fine as possible at the most critical regions, while allowing it to be coarse at the rest of the model. Obviously, the most critical areas are in the vicinity of the contact line and the fillet between the flank and the root surface of the gears. At first approximation, one could consider making a fine mesh at the entire flank and fillet areas. But this approach would result in extremely high element numbers. Aiming at keeping the element number as low as possible — while maintaining the accuracy of the results — the fine mesh should be located near the instantaneous contact line and not over the entire active flank. Although it is possible to make this mesh manually, one should consider the effort and time needed. On one hand, even in simple planetary systems like the one considered in the current study, there are as many as 12 gear teeth contacting each other. On the other hand, to meet the requirements of a quasi-static analysis, many successive snapshots need to be generated and solved. Obviously, this process needs to be automated.

Models were created using the ANSA pre-processor's dedicated tools for automatic mesh generation ("Batch Mesh") in combination with the mesh refinement boxes ("BCBOX") (Ref. 2). Batch meshing makes it possible for the user to define both the meshing requirements and the quality criteria for every region of the model that should be automatically generated for every snapshot. The mesh refinement boxes are hexahedral boundaries defined by the user that enclose the regions, which must be extremely fine-meshed; in the current model these are: first, the contact lines, which are defined as the intersections of the active flanks and the plane of action of each mesh, and second, the fillet at the roots of the mating teeth. The decisive advantage of using these boxes is that they are fixed to the planet carrier and therefore they do not need to be repositioned when creating a new snapshot. Figure 2 shows the *BCBOX* of the plane of action of a planet — ring gear engagement. The final step is to introduce manufacturing deviations. In the current development stage, the pitch error is considered by modifying the tooth thickness in a way previously introduced (Ref. 1). The *ANSA* "morphing" tool is employed, thus allowing the introduction of pitch

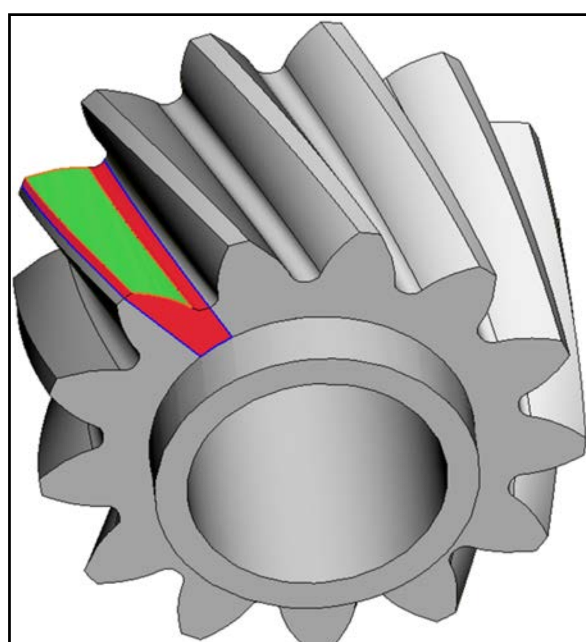

Figure 3 Modification of the tooth thickness by morphing. Only the active flank and the surrounding marked surface are affected, while the bounds are fixed.

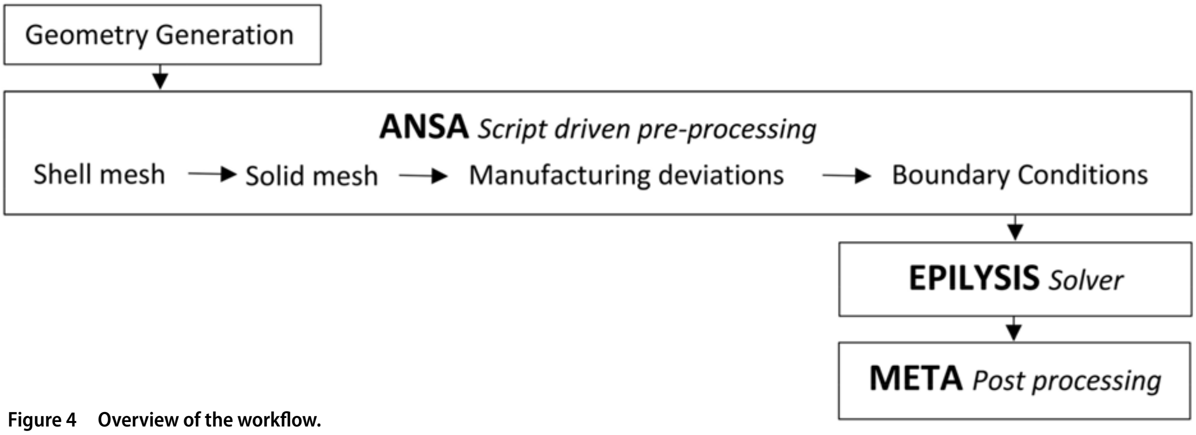


**Figure 4 Overview of the workflow.**

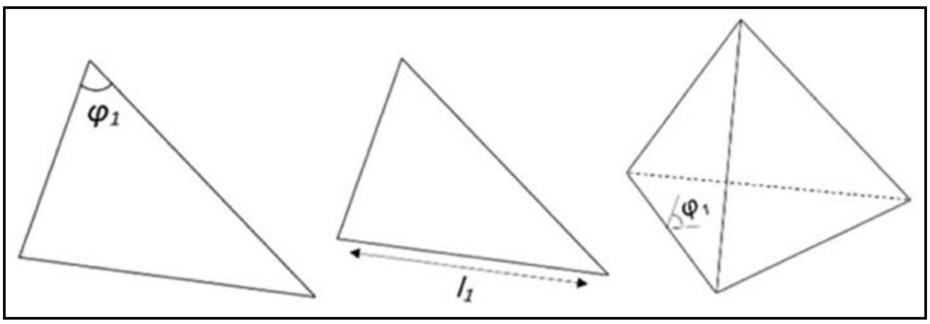


**Figure 5 Mesh quality parameters.**

error to the model (Fig. 3).

The third task is to apply the boundary conditions. Once the mesh has been automatically generated, contact pairs need to be defined. A contact pair includes all entities that might contact each other during the solution process. This step should also be automated because at every snapshot the entire mesh is regenerated and therefore the previously defined contact pairs are outdated. A custom script featuring the *ANSA* generic entity builder ("*GEB_SB*"), combined with the refinement boxes, is employed to define the contact pairs. In this way it is assumed that only the active flanks of the engaged gears may contact each other; contacts of other surfaces are excluded from the analysis. Since the model is quasi-static, the rotational speed is not considered. Therefore the planet carrier is assumed to be stationary. Torque is applied to the sun gear. Depending on the case solved, it is either allowed to rotate around the fixed planetary system axis or to rotate around its own axis and center itself between the planets. The planets are mounted on the carrier by rolling element bearings and are allowed only to rotate around the axis of their pins. Finally, the ring gear is fixed.

## Solution Scheme and Results

Figure 4 shows an overview of the workflow. The entire analysis is carried out using the BETA CAE Systems suite, consisting of *ANSA* pre-processor, *EPILYSIS* solver and *META* post-processor. As mentioned above, quasi-static conditions are assumed.

To obtain a good quality mesh, target values for the minimum/maximum element angle $\varphi_1$, as well as for the maximum element length $l_1$, were set. Figure 5 shows the above quality parameters. For the entire model, $\varphi_1$ was limited in the range 45°–75°.

In the root fillet area, the target element length was set at 0.2 mm and the allowable range 0.1–2 mm. Along the contact line the target element length was set 0.08 mm and the allowable range 0.04–1.5 mm. In these areas the so-called "solids structural mesh algorithm" was applied, which resulted in

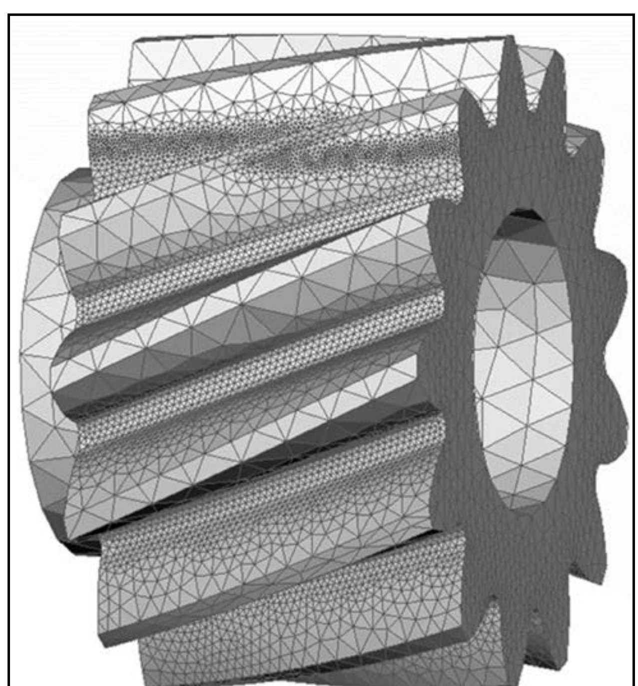

**Figure 6 3-D FE mesh of the sun gear.**

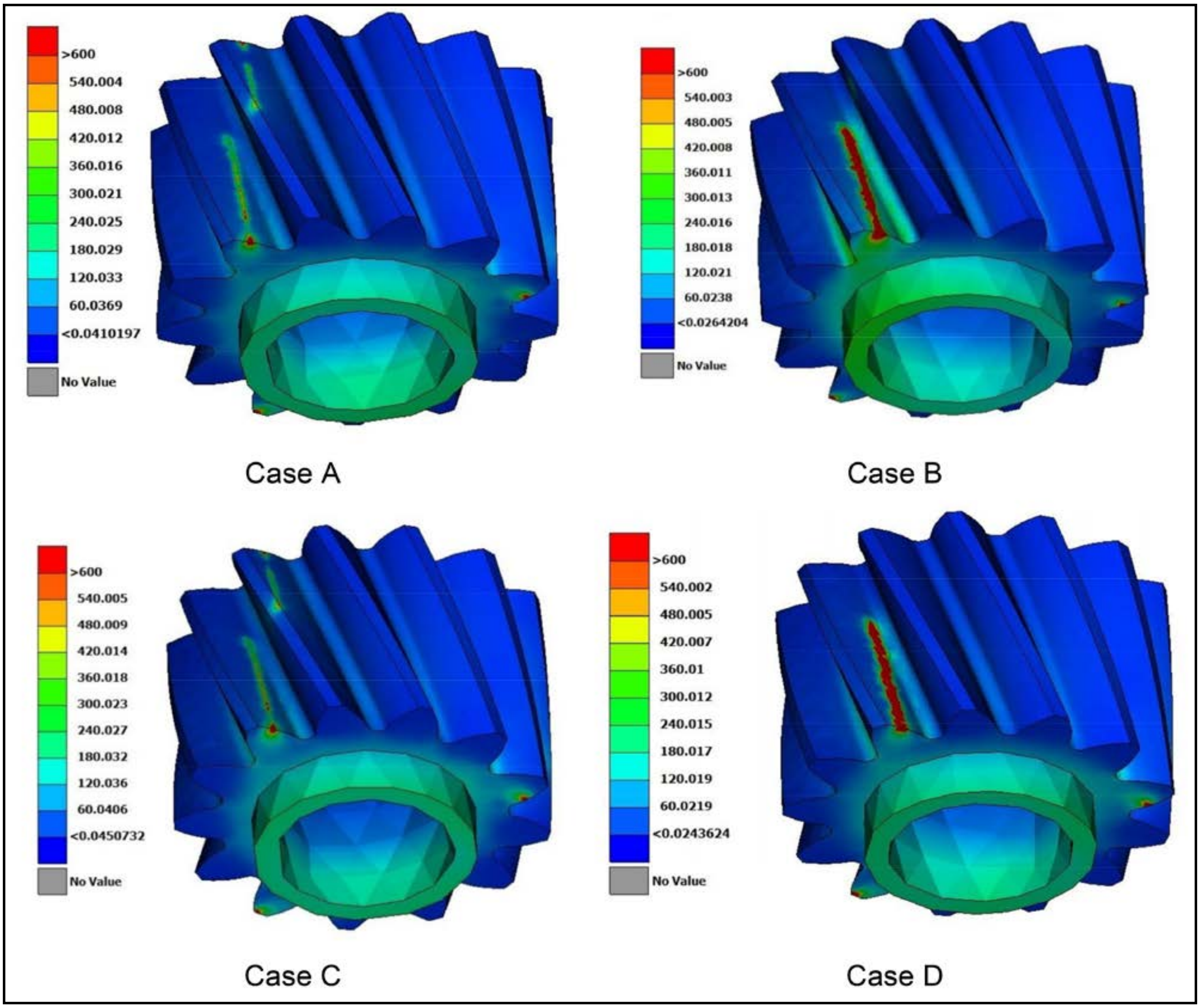


**Figure 7** **Resulting von Mises stress field of the above cases. The impact of the pitch error on the contact stress is obvious; the favorable effect of a free sun can be recognized at the fillet area.**

a fine mesh of uniform density. In the remaining areas of the model, a special algorithm was implemented in order to generate a smooth yet fast transition from the fine to the coarse mesh. The growth rate was chosen 1.6. Figure 6 shows the 3-D FE model of the sun gear. It should be emphasized that the mesh in the unloaded areas is almost 20 times coarser than along the contact line. The following 4 cases were solved:

Case A: Geometrically perfect gears; sun gear allowed only to rotate around the axis of the planetary system.

Case B: Sun gear with 20-µm pitch error; sun gear allowed only to rotate as in case A.

Case C: Geometrically perfect gears, sun gear allowed to center itself between the planets. It can move as needed, meaning that its axis may reach an off-center and oblique position relative to the ring gear axis. Only movement in the axial direction is restricted. In real planetary gear systems, this model corresponds to a sun gear driven by a double-articulated joint.

Case D: Sun gear with 20-µm pitch error; sun gear floating as in case C.

In all cases the torque applied to the sun gear was 15 Nm, which results in approximately 1250 N/mm$^2$ contact stress.

Figure 7 shows the resulting von Mises stress field of the above cases; the impact of the pitch error on the contact stress is obvious. The favorable effect of a free sun can be recognized at the fillet area.

Load sharing is the most interesting result of the analysis; Table 1 shows the results obtained.

It should first be noticed that even in geometrically perfect planetary gear systems, the torque distribution is not uniform (case A). This should be attributed to the fact that the meshing stiffness varies along the line of action. Since the meshing points of the sun differ with each planet, it is evident that

**Table 1 Load sharing results**

| | Case A | Case B | Case C | Case D |
|---|---|---|---|---|
| Planet 1 | 0.9065 | 1.6993 | 0.9970 | 1.0011 |
| Planet 2 | 0.9245 | 0.5439 | 1.0024 | 1.0017 |
| Planet 3 | 1.1688 | 0.7566 | 1.0000 | 0.9993 |
| Load mesh load factor $K_{\gamma}$, | 1.1688 | 1.6993 | 1.0024 | 1.0017 |
| Overload | +16.88 % | +69.93 % | +0.24 % | +0.17 % |

differences in the load distribution will occur. When the sun can float (case C), torque is distributed almost uniformly. As expected, pitch errors have a strong impact on the load distribution (case B). Again, a self-aligning sun gear enhances significantly the torque distribution (case D). Figure 8 shows the deformation of the self-aligned sun in case D. It should be noticed that its axis is oblique relative to carrier axis.

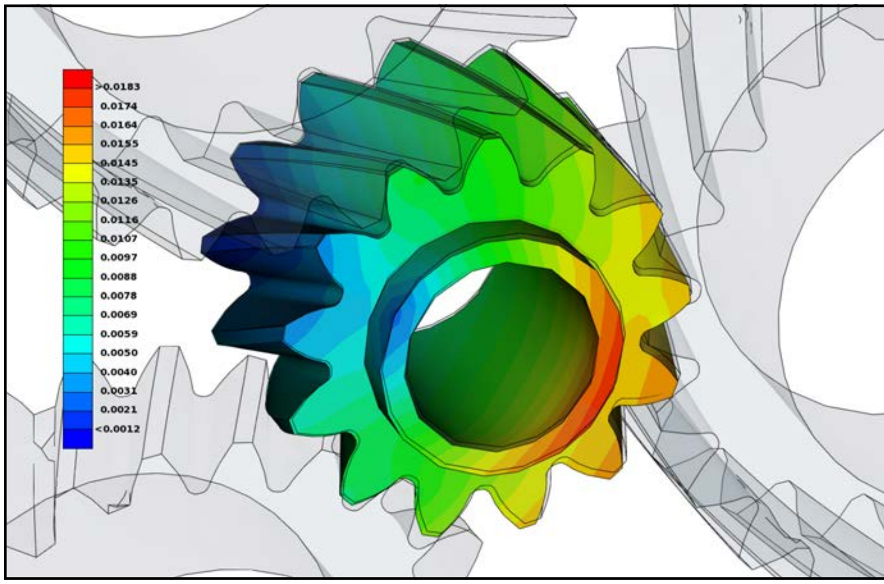


**Figure 8** Deformation of the sun gear in case D (scale factor: X 20; non-deformed gears are shown in transparent grey).

## Conclusions

A simple planetary gear train consisting of helical gears was analyzed in the current study using 3-D FE modeling.

The analysis was made possible by employing advanced meshing techniques, which helped to keep the DOFs manageable, as well as scripting, which automated the procedure — allowing easy generation of models for successive snapshots.

Results demonstrate the beneficial effect of a floating sun and the impact of manufacturing deviations.

Next steps of the research will include the analysis of a floating ring gear, as well as the impact of pin position errors. **PTE**

## References

1. Mihailidis, A, I. Panagiotidis, E. Bouras and E. Athanasopoulos. "Finite Element Method Based Analysis of Planetary Gear Systems Considering Backlash and Manufacturing Errors," *International Conference on Gears* 2015, 5–7 October, Munich, Germany, VDI Berichte 2255, 2015, pp. 79–88.
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